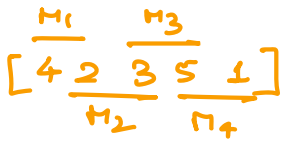
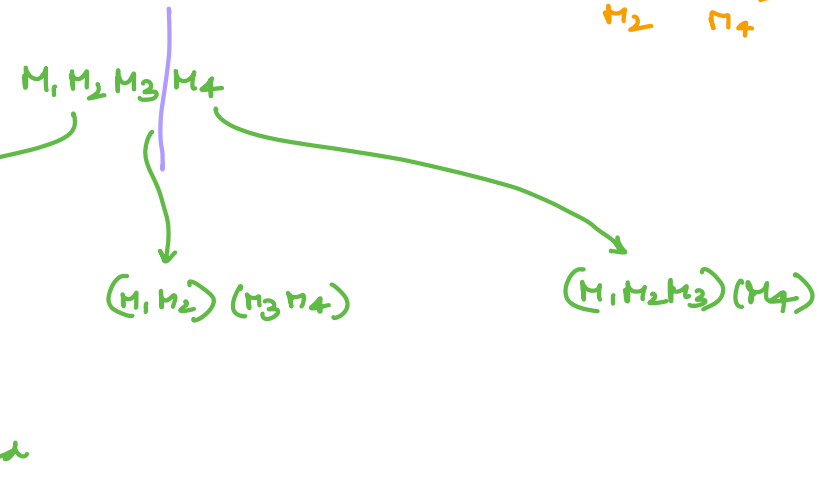
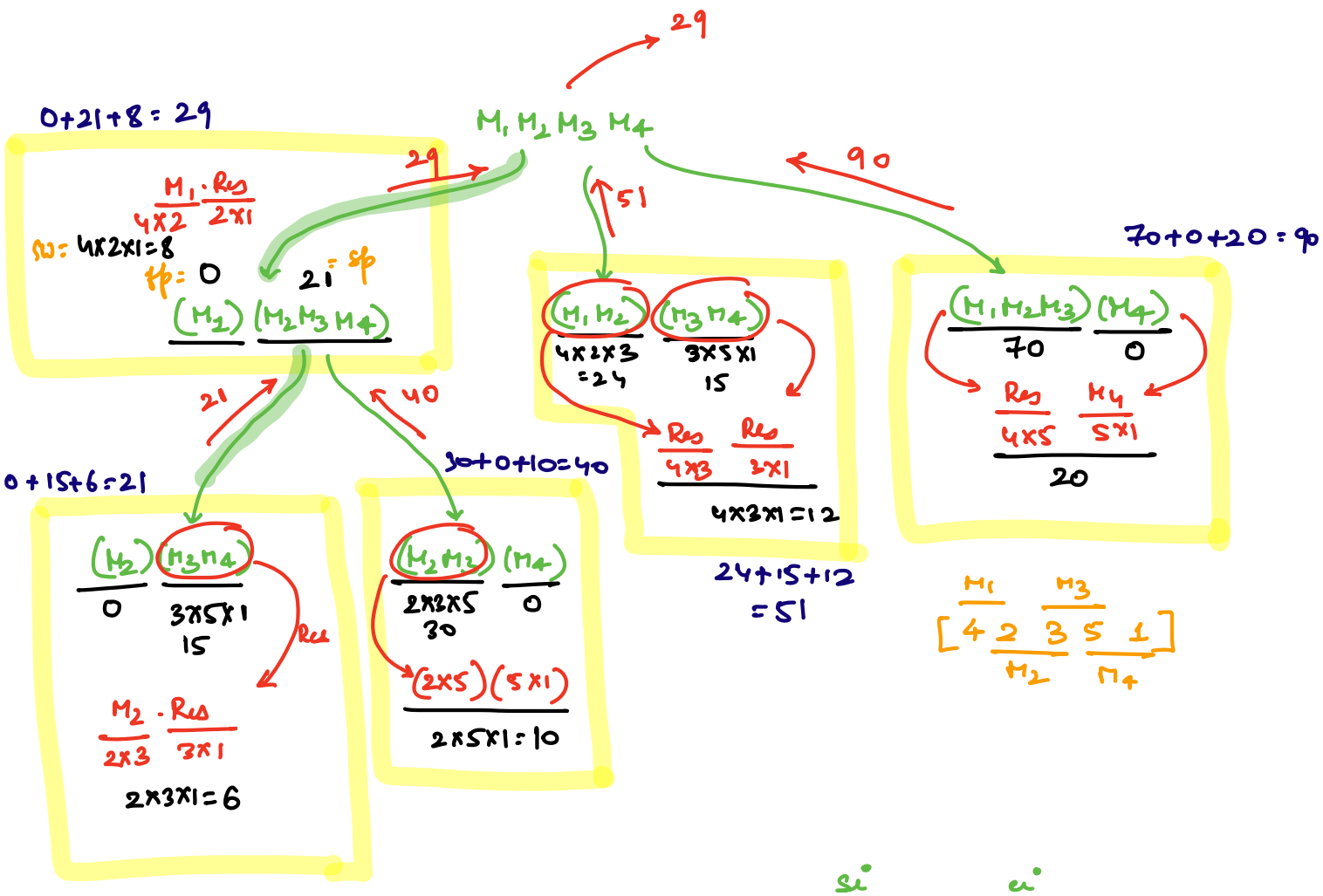
Partition DP — MCM, Palindrome Partitioning, Mixtures

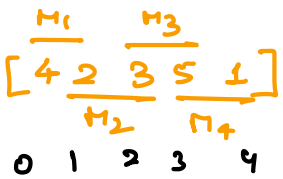


Recursion:



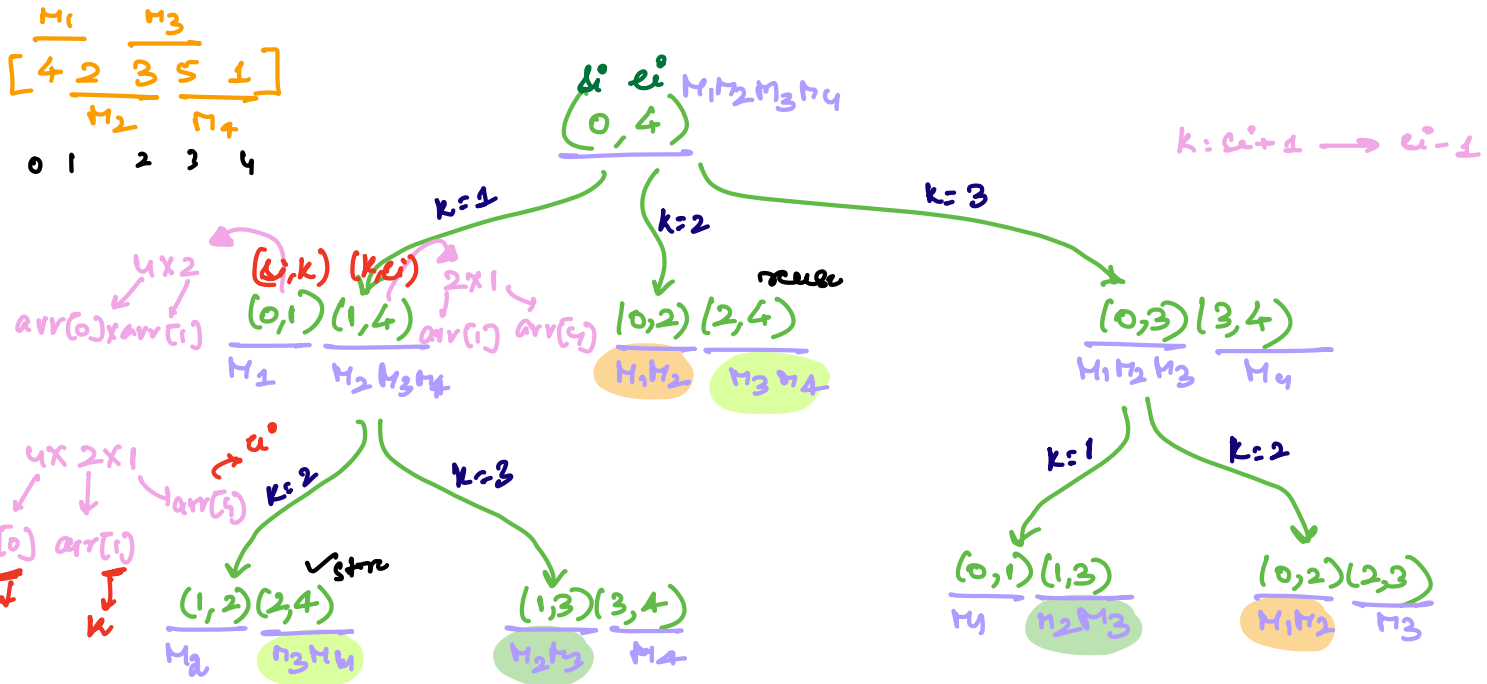
Rec Call Return:





	si	ei
M1 M2 M3 M4	0	4
M2 M3 M4	1	4
M1 M2 M3	0	3
M1 M2	0	2

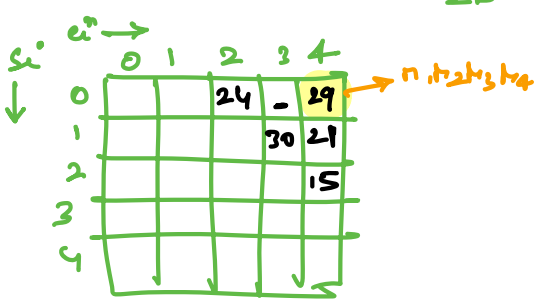
M_2M_3	1	3
M_3M_4	2	4
M_1	0	1
M_3	2	3



1. Recursion code
2. Code taking lot of time for bigger input
3. RT notice : duplicacy
4. DP : TD Easy : Store (Memoization)

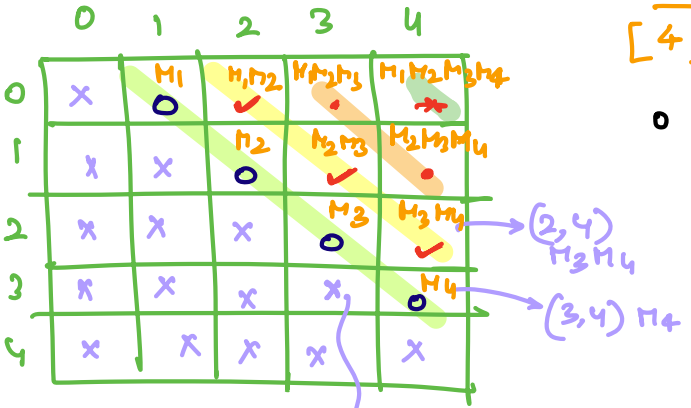
Strg 1D, 2D, 3D ? si, ei

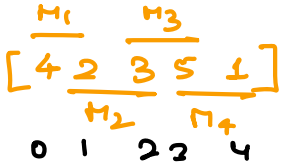
2D



Bottom up DP (Tabulation)

1. Strg size?
   2D





3,3

2. Cell meaning ?

3. TD BC return → BU fill work start

if ($s_i + 1 == e_i$) } m_1, m_2, m_3, m_4 : return 0
return 0

4. filling direction

slide = 1	
m_1	0,1
m_2	1,2
m_3	2,3
m_4	3,4

(s_i, e_i)

slide = 2	
$m_1 m_2$	0,2
$m_2 m_3$	1,3
$m_3 m_4$	2,4

(2)

slide = 3	
$m_1 m_2 m_3$	0,3
$m_2 m_3 m_4$	1,4

(3)

slide = 4	
$m_1 m_2 m_3 m_4$	0,4

(4)

slide, s_i

$e_i = s_i + slide$

n = 5

slide	s_i	5-slide	5-slide-1
1	3	4	3
2	2	3	2
3	1	2	1
4	0	1	0

BU:

Time Complexity

$\frac{n^2}{2}$ elements fill

1 element → k loop → n

$\frac{n^2}{2} \times n = O(n^3)$

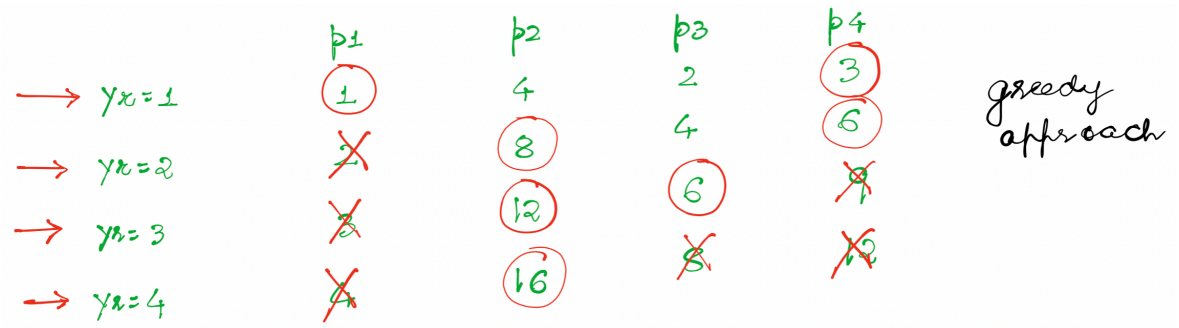
Space Complexity: $O(n^2)$

Wine Problem:

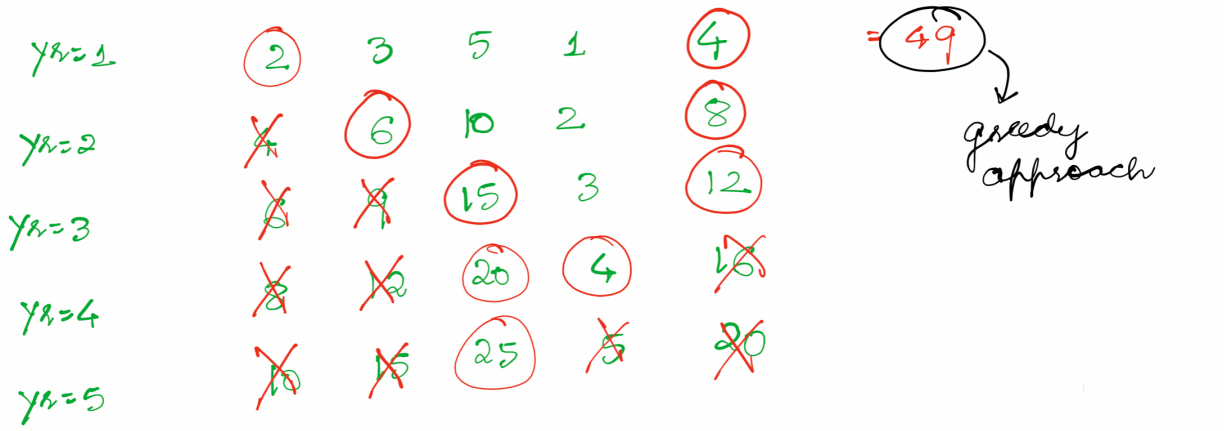
Imagine you have a collection of N wines placed next to each other on a shelf.
For simplicity, let's number the wines from left to right as they are standing on the shelf with integers from 1 to N, respectively.
The price of the ith wine is pi. (prices of different wines can be different).
Because the wines get better every year, supposing today is the year 1, on year y the price of the ith wine will be y*pi, i.e. y-times the value of initial year.

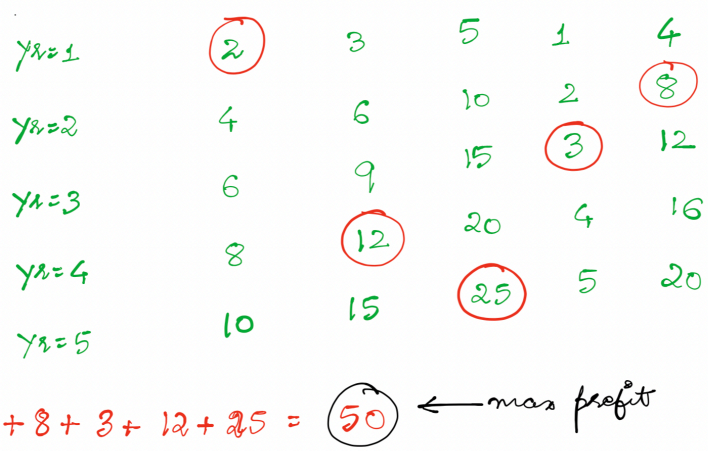
You want to sell all the wines you have, but you want to sell exactly one wine per year, starting on this year.
One more constraint - on each year you are allowed to sell only either the leftmost or the rightmost wine
on the shelf and you are not allowed to reorder the wines on the shelf (i.e. they must stay in the same order as they are in the beginning).
You want to find out, what is the maximum profit you can get, if you sell the wines in optimal order?

Example :
If prices are : p1=1, p2=4, p3=2, p4=3
Then maximum profit is 1 * 1 + 3 * 2 + 2 * 3 + 4 * 4 = 29.

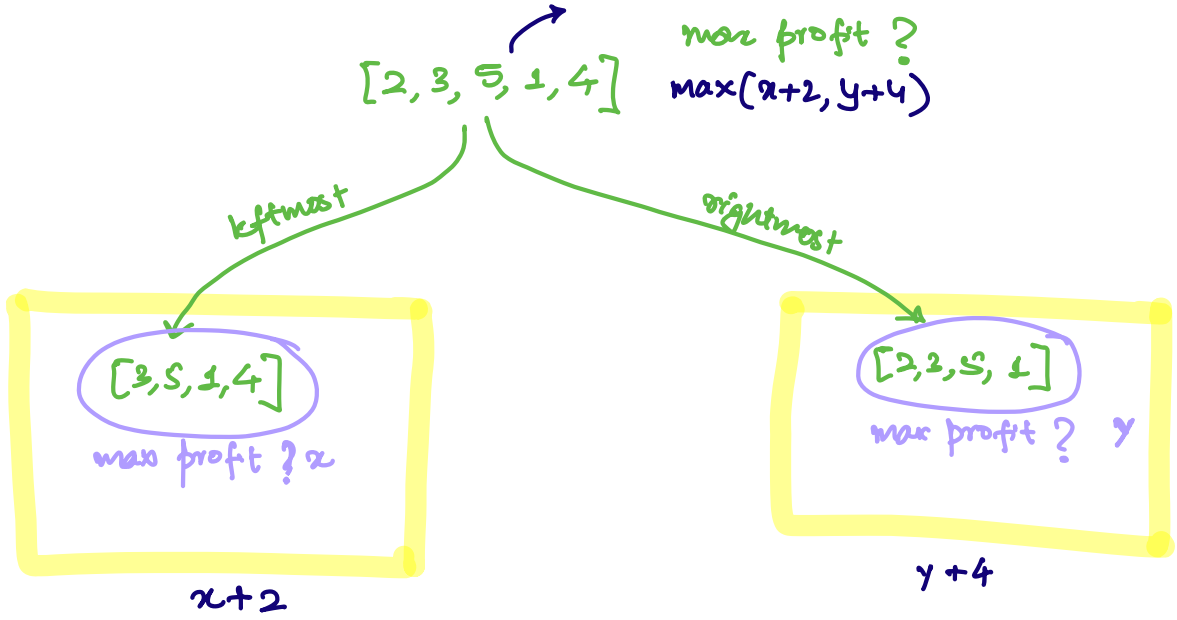


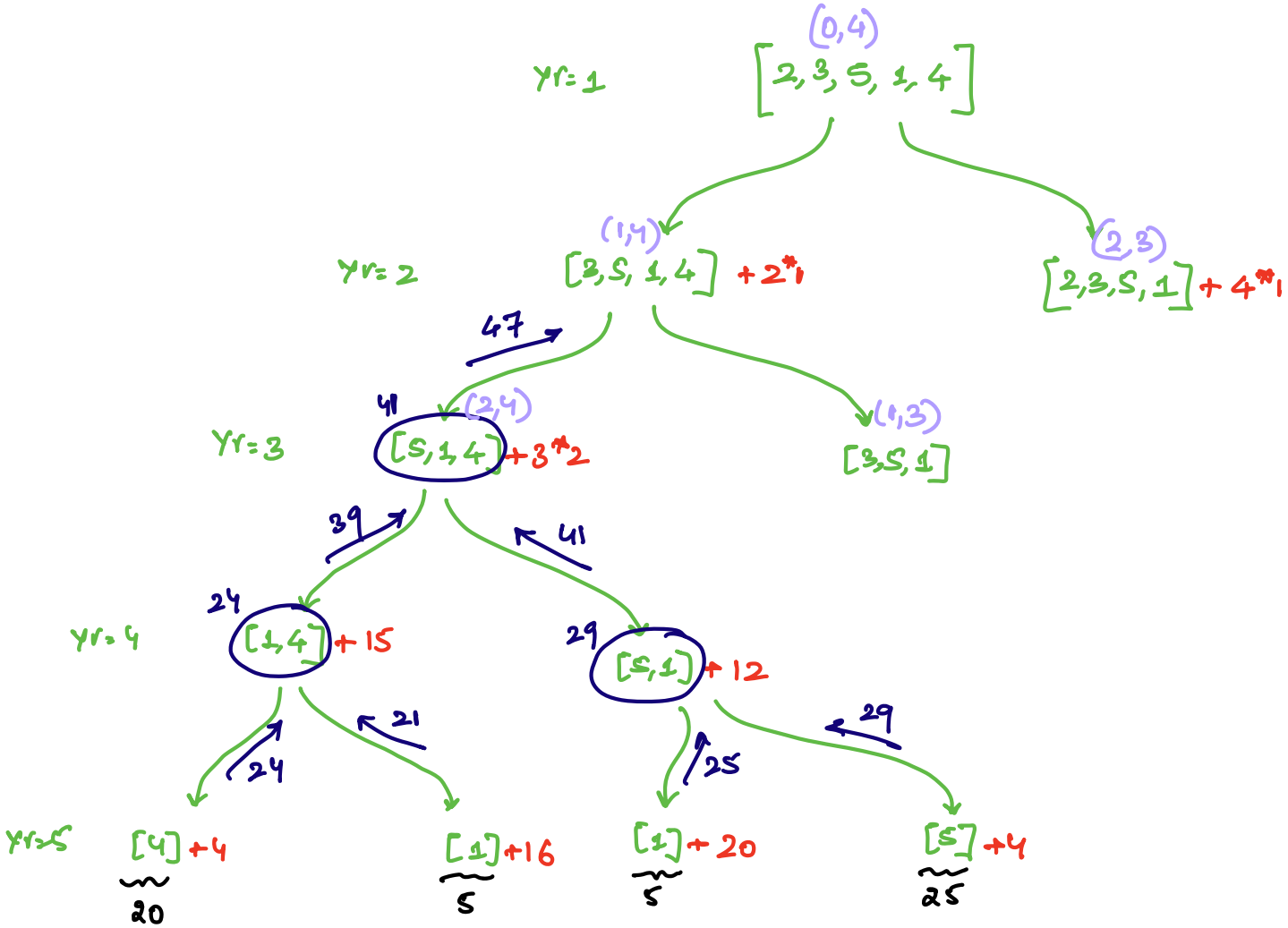
If the prices of the wines are: p1=2, p2=3, p3=5, p4=1, p5=4 then total profit is ?





Recursion





[2, 3, 5, 1, 4]
0 1 2 3 4

meaning	si	ei
[2,3,5,1,4]	0	4
[3,5,1,4]	1	4

[3,5,1]	1	8
[5]	2	2

Year can be derived through s_i & c_i

[2, 3, 5, 1, 4]

[_, _] yr = 4

[_, _, _] yr = 3

[_, _, _, _] yr = 2

s_i, c_i

bottles left = $c_i - s_i + 1$

bottles sold = $n - (c_i - s_i + 1)$

yrs already passed = $n - (c_i - s_i + 1)$

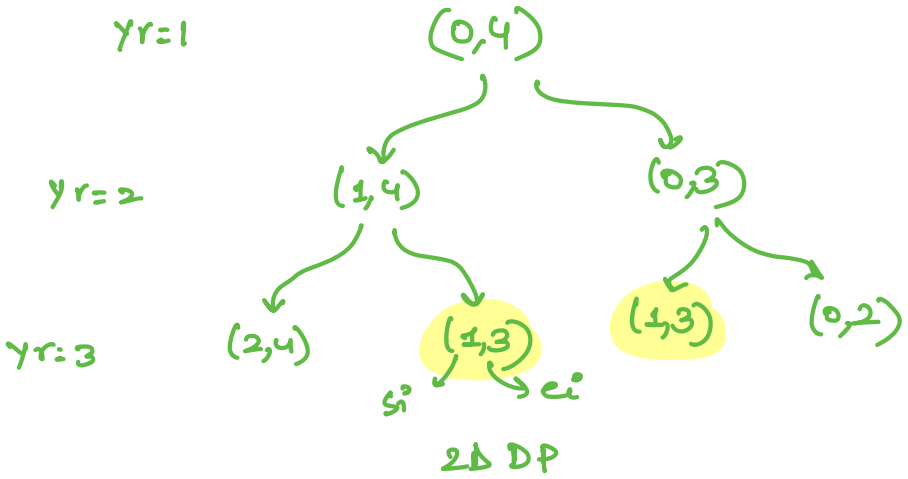
present yr = $n - (c_i - s_i + 1) + 1$

$$yr = n - c_i + s_i$$

0 1 2 3 4
2, 3, 5, 1, 4

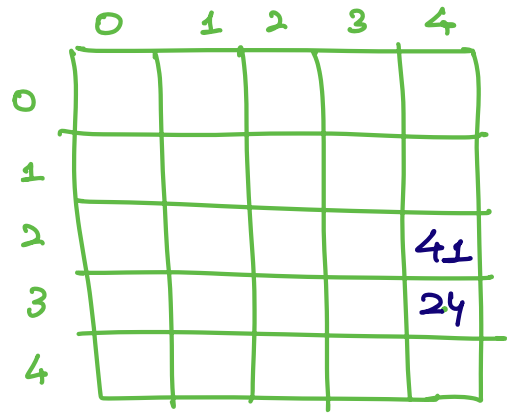
$s_i \leftarrow 2\ 3 \rightarrow c_i$
[5, 1]

left = 3 - 2 + 1 = 2
sold = n - 2 = 5 - 2 = 3 bottles sell
yrs passed by = 3 yrs
present yr = 4



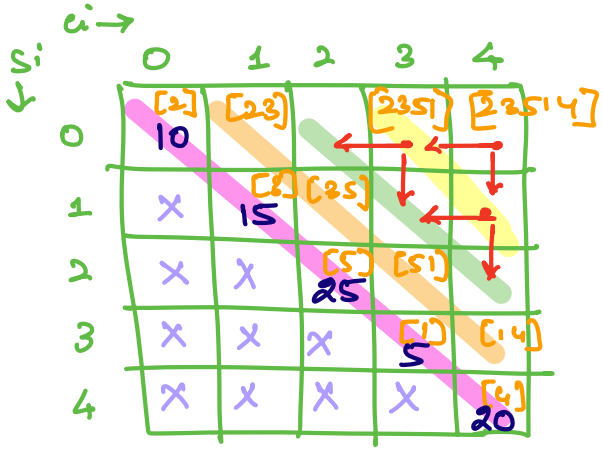
2D DP

0 1 2 3 4
2, 3, 5, 1, 4



Bottom up:

- Size ? 2D
- Cell meaning
- TD BC return
  ↓
  BU fill start
- filling direction



si > ci : useless

0 1 2 3 4
2, 3, 5, 1, 4

si ci	si ci	si ci	si ci	si ci
00	01	02	03	04
11	12	13	14	
22	23	24		
33	34			
44				

slide=0	slide=1	slide=2	slide=3	slide=4
si = 0→4	si = 0→3	si = 0→2	si = 0→1	si = 0→0

slide	si	s. slide	s-slide-1
0	4	5	4
1	3	4	3
2	2	3	2
3	1	2	1
4	0	1	0

Tentative schedule:

26 Oct: sunday: 2-3hrs

28 Oct: Tuesday: 1hr offline (DP)

30 Oct: Thursday: Class, Papers

31 Oct: Friday: Class

3 Nov: Monday: Test Graphs

4 Nov: Tuesday: Test DP